

## CORRIGÉ DU DM N°10

**Correction 1. réponses**

1.  $z_1 = 3\sqrt{2}e^{i\frac{3\pi}{4}}$      $z_2 = 4\sqrt{3}e^{-i\frac{\pi}{3}}$      $z_3 = 4e^{i\frac{\pi}{3}}$     et     $z_4 = 2\sqrt{2}e^{-i\frac{\pi}{4}}$
2.  $z_1 \times z_2 = 12\sqrt{6}e^{i\frac{5\pi}{12}}$      $\frac{z_4}{z_1} = \frac{2}{3}e^{i\pi}$      $\frac{z_2}{z_4} = \sqrt{6}e^{-i\frac{\pi}{12}}$     et     $(z_3)^6 = 4^6e^{i2\pi}$ .

**Correction 2.**

1.  $|z| = \frac{|1 + \sqrt{2} + i|}{|1 + \sqrt{2} - i|} = \frac{|1 + \sqrt{2} - i|}{|1 + \sqrt{2} - i|} = 1$  car un nombre et son conjugué ont le même module :

$$\boxed{|z| = 1}.$$

2. 
$$z = \frac{(1 + \sqrt{2} + i)^2}{|1 + \sqrt{2} - i|^2}$$

$$= \frac{(1 + \sqrt{2})^2 + 2i(1 + \sqrt{2}) + i^2}{(1 + \sqrt{2})^2 + 1^2}$$

$$= \frac{1 + 2\sqrt{2} + 2 + 2i(1 + \sqrt{2}) - 1}{1 + 2\sqrt{2} + 2 + 1}$$

$$= \frac{2\sqrt{2} + 2 + 2i(1 + \sqrt{2})}{4 + 2\sqrt{2}}$$

$$= \frac{2(\sqrt{2} + 1)}{4 + 2\sqrt{2}}(1 + i)$$

$$\text{Or } 4 + 2\sqrt{2} = 2 \times 2 + 2\sqrt{2} = 2 \times \sqrt{2}^2 + 2\sqrt{2} = 2\sqrt{2}(\sqrt{2} + 1).$$

$$\text{Donc } z = \frac{2(\sqrt{2} + 1)}{2\sqrt{2}(\sqrt{2} + 1)}(1 + i) = \frac{1}{\sqrt{2}}(1 + i) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i.$$

3.  $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \cos(\frac{\pi}{4}) = \sin(\frac{\pi}{4})$  donc  $\arg(z) = \frac{\pi}{4} [2\pi]$  donc  $z = e^{i\frac{\pi}{4}}$  (car  $|z| = 1$ ).  
 Donc  $z^{2023} = e^{i\frac{2023\pi}{4}}$ .  
 Or  $2023 = 2024 - 1 = 8 \times 253 - 1$  donc  $\frac{2023\pi}{4} = 253 \times 2\pi - \frac{\pi}{4} = -\frac{\pi}{4} [2\pi]$ .  
 Donc  $\boxed{z = e^{-i\frac{\pi}{4}}}$ .

**Correction 4. Réponses.**

$$A = 1 \quad B = \frac{1}{81} \quad C = \sqrt{5} \quad D = -1 \quad E = -\frac{1}{2}.$$